## Calculating pi by hand for 100 digits

Machin in 1706 created the formula and used it to calculated 100 digits, and since then has been known as Machin's Formula alone with other similar atan functions. Other individual also used the same formula are Clausen in 1847 with 125, Rutherford in 1853 with 440 followed by William Shanks in 1853 with the first 530 and then 609 finally in 1873 his 709 which was reduced to 707 digits. Each of the four have seen some internal numbers have never been publish, like the sum of the positive and negative terms not even in William Shanks book which showed each term but no sub term and sub sums.

In my hand calculation of 40 digits it took 16 pages while my 100 digits took 108 pages, I was not as conservative as with the 40 digit calculation. The number of pages goes with the square of the ratio of the number digits, there are that many time so terms and each term has that number of digits. So 100 versus 40 a ratio of 2.5 would be 16\*2.5\*2.5=100 pages while it took William Shanks 709 to 100 is 7.09 which 108\*7.09\*7.08=5429 pages. In modern terms would be 11 reams of paper which is over a case of paper. A ream of paper is about 2 inches thick and 11 reams would be a pile 22 inches tall. Using the 16 pages for the 40 digit would produce 17.725\*17.725\*16= 5027 pages for William Shanks, the real value may be between the two numbers. I have never seen any place that tells just how much paper he did use. The big question is any or all of the detailed calculations even exist I would have hoped that they were give to some group for historical keep sake. Even Srinivasa Ramanujan wife donated his note books to the math world so they were not lost. Some of Machin's papers have been saved but there is no reference where his calculations of PI were saved.

The breakdowns of how many pages my calculations required are as follow. The 1/5 power sub terms required 3 pages while the full terms were another 47 pages. The 1/239 power sub terms required 27 pages while the full terms were another 21 pages. Finally the summary pages were another 10 pages which brings the total to 108 pages. I spent 23 day doing the calculations from September 22 2014 through October 14, 2014.

I have looked to see if anyone else has truly done a "hand calculation" and showed the work. The only one are the three samples that I have done the 20, 40 and 100 digits although I did not feel like adding all 108 pages, may be some day they will be added as another file. The main purpose for this was to find out how many page for 100 would supply an answer page required by William Shanks, and it did.

I have included only the 10 final summary pages where the positive and negative terms were brought together and the third and fifth terms from the 239 powers just to show you the work required for each term. Note the numbers in the upper left box with the numbers "1-50" or "51-100" this will tell you what digits are on that page as it take two pages to handle the 100 digits. As the work goes further along it will take only 1 page the 51-100.

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