

Calculating pi by hand for 100 digits

Machin in 1706 created the formula and used it to calculate 100 digits, and since then has been known as Machin's Formula along with other similar atan functions. Other individuals also used the same formula are Clausen in 1847 with 125, Rutherford in 1853 with 440 followed by William Shanks in 1853 with the first 530 and then 609 finally in 1873 his 709 which was reduced to 707 digits. Each of the four have seen some internal numbers have never been published, like the sum of the positive and negative terms not even in William Shanks' book which showed each term but no sub term and sub sums.

In my hand calculation of 40 digits it took 16 pages while my 100 digits took 108 pages, I was not as conservative as with the 40 digit calculation. The number of pages goes with the square of the ratio of the number of digits, there are that many times so many terms and each term has that number of digits. So 100 versus 40 a ratio of 2.5 would be $16 \times 2.5 \times 2.5 = 100$ pages while it took William Shanks 709 to 100 is 7.09 which $108 \times 7.09 \times 7.09 = 5429$ pages. In modern terms would be 11 reams of paper which is over a case of paper. A ream of paper is about 2 inches thick and 11 reams would be a pile 22 inches tall. Using the 16 pages for the 40 digit would produce $17.725 \times 17.725 \times 16 = 5027$ pages for William Shanks, the real value may be between the two numbers. I have never seen any place that tells just how much paper he did use. The big question is any or all of the detailed calculations even exist I would have hoped that they were given to some group for historical keep sake. Even Srinivasa Ramanujan's wife donated his note books to the math world so they were not lost. Some of Machin's papers have been saved but there is no reference where his calculations of PI were saved.

The breakdowns of how many pages my calculations required are as follows. The $1/5$ power sub terms required 3 pages while the full terms were another 47 pages. The $1/239$ power sub terms required 27 pages while the full terms were another 21 pages. Finally the summary pages were another 10 pages which brings the total to 108 pages. I spent 23 days doing the calculations from September 22, 2014 through October 14, 2014.

I have looked to see if anyone else has truly done a "hand calculation" and showed the work. The only ones are the three samples that I have done the 20, 40 and 100 digits although I did not feel like adding all 108 pages, maybe some day they will be added as another file. The main purpose for this was to find out how many pages for 100 would supply an answer page required by William Shanks, and it did.

I have included only the 10 final summary pages where the positive and negative terms were brought together and the third and fifth terms from the 239 powers just to show you the work required for each term. Note the numbers in the upper left box with the numbers "1-50" or "51-100" this will tell you what digits are on that page as it takes two pages to handle the 100 digits. As the work goes further along it will take only 1 page the 51-100.

10 105 items

1 to 50																				
Carry 10	00000	00001	10121	22222	24243	43324	4545	44555	67566	66877										
Carry 7	20000																			
4	00006	40008																		
5		00568	88888	89888	88888	88888	88888	88888	88888	88888										
9			63015	38461	53846	15384	601538	46153	84615	38461										
13			00077	10117	64705	88235	29411	26470	58823	52941										
17				09986	43909	52380	96238	09523	80952	38095										
21																				
25					00013	42177	28													
29					01851	27900	68965	51724	13793	10344										
33					00002	60301	04824	24242	42424	24242										
37						00371	45663	10054	06406	40540										
41							53634	71355	00487	80487										
45							00078	18749	35307	37777										
49								11488	77455	96186										
53								00016	99471	55749										
57									02528	33663										
61									00003	78007										
65										00567										
69																				
73																				
77																				
81																				
85																				
89																				
93																				
97																				
101																				
105																				
109																				
113																				
117																				
123																				
125																				
129																				
133																				
137																				

Total 120006 40569 51981 47467 95281 61463 48243 08667 90157 75954

1 to 50	Neg Terms									
Carry 10										
Carry 1	00000	00000	00000	01001	00010	11112	11100	01212	22212	12212
3		00244	16591	78708	38036	27411	89230	12459	51820	44392
7				00320	71306	57784	69471	70443	58353	10750
11						00625	50445	09921	56534	92927
15								01405	85401	86257
19										09401
23										
27										
31										
35										
39										
Total		00244	16591	79029	09342	85822	09146	94230	52116	59729
Carry 10										
Carry 1	00000	00000	00000	10100	00112	20011	02012	11132	22221	32223
1	00718	41004	18410	04184	10041	84400	41841	00418	41004	18410
5			00256	47231	44246	47365	20520	71108	82995	23979
9					00436	69315	24403	91897	49849	75066
13							00926	58216	24156	09570
17									02171	63464
21										
25										
29										
33										
37										
41										
Total	00418	41004	18666	51415	54725	00781	37692	21641	00175	90492

51 to 100	21207	68516	13315	31486	52581	36969	60844	84532	90705	87072
	04184	10041	84100	41841	00418	41004	18410	04184	10041	84100
	0		49	4244			2589	24		
	2		45	6968			2284	87		
	84		3	72761			304	101		
	21		3	42706			285	605		
	631			30035	0		18	7968		
	242			28560	5		17	1363		
	38984			1474	50					
	09847			1142	42		1	66054		
	39132	1		332	084		1	14242		
	342726			285	605			51812	1	
	486450			46	4291			67408	9	
	456968			45	6968			403	200	
	294820				76239			399	947	
	285605				57121			3	35341	
	9	2154			21117	4		2	85605	
	5	7121			17136	3			49736	8
	3	50331			3921	11			464004	
	3	42726			3427	26			399847	
		76058			553	850			41	5710
		6712	1		514	089			39	9847
		1893	74		39	7610			1	58630
		1713	63		34	2726			1	14242
		180	111		5	48844				44388
		171	363			14069				
		8	7480			34755				
		5	7121			34272	6			
		3	03590			482	584			
		2	85605			458	968			
			17985	4		25	6161			
			17136	3		22	8484			
			849	11		2	76770			
			571	21		2	284821			
			277	908			48286	D		
			228	484			45696	8		
			49	424			2589	2		

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51 to 100	99156	54851	42970	41920	33654	25467	63833	98128	26338	66189
	21207	68516	13315	31486	52581	36969	60844	84532	90705	87072
	2		11	6158			4642	94		
	9		11	4242			4569	68		
	31			19166	5		73	265		
	89			17136	3		57	121		
	422			2030	22		16	1443		
	121			1713	63		11	4242		
	3010			316	595		4	72012		
	5605			285	605		4	56968		
	74057			309908	28		15044	9		
	42726			28	5605		11424	2		
	31331	6		2	43031		3620	70		
	28560	5		2	28484	3	3427	26		
	2771	18		2	14543	2	193	447		
	2284	84		11424	11424	2	171	363		
	486	345		3123	16		22	0840		
	456	968		2856	05		17	1363		
	29	3771		267	119		4	94775		
	28	5205		228	484		4	56968		
		81666		38	6356		37807	8		
		57121		34	2726		34272	6		
		24545	1	4	36309		3535	27		
		22848	4	3	99847		3427	26		
		1698	73		36462	6	108	010		
		1142	42		34272	6	57	121		
		554	313		2190	00	50	8897		
		514	089		1713	63	45	6968		
		40	2241		476	378	5	19292		
		39	9847		456	968	5	14089		
			23970	3	19	4104		5203		
			22848	4	17	1363				
			109691		2	27414				
			571	71	1	21363				
			525	704		56091	8			
			514	089		51408	9			
			11	615		4642	9			

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539